Aggregate Demand Model versus Conventional Regression Analysis

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Abstract

The importance of the demand curve cannot be neglected for the business managers who strive to maximize their profits. In market environment it is important to set the right price for the product so that to successfully compete with the rivals. The current paper introduces new method of analysis of demand internal structure and compound nature depended on the contribution of various groups of customers. Direct and inverse problem of estimation of elementary and aggregate demands parameters are defined. Aggregate demand structure is represented as a multidimensional dummy variables regression model. The theoretical results are verified by means of corresponding numerical example.

Keywords: market demand, aggregate demand, internal structure, price, regression analysis
JEL Classification: C13, C18, C51, C53

1. Introduction

Demand estimation problem arises each time when there is need f or forecasting of the sales volume, optimal price settlement for profit maximization, or for empirical studies of the market for demand. Because there is a relationship between price and quantity demanded, it is important to understand the impact of pricing on sales by estimating the demand curve for the product. Market demand for the product can be obtained by survey or experiments can be performed at prices above and below the current price in order to determine the price elasticity of demand.

Alfred Marshall (1890) approached market demand curve from individual demand curves obtained by marginal utility principle, some other economists used indifference curve analysis to serve the same goal. However the applications of these two methods are difficult so econometric methods have been developed in recent years. There are researchers (for example, Kayser (2000), Mannering and Winston (1985), Archibald and Gillingham (1980)) that apply multiple regression equation to historical statistical data to form estimated market demand curve, and there are others (for example, Afriat(1967), Diewert(1973, 1985), Lansburg(1981), Varian(1982, 1983, 1985), Chavas and Cox(1997)) which use nonparametric methods to analyze behavior of consumers with observed data without specifying function form of the preferences or/and demand function.

One of the recent developments in demand estimation techniques, the BLP method of structural demand estimation using the random-coefficients logit model, is enhanced by the Rasmusen (2007). In this method the demand estimates from the individual demand coefficients of each customer. So that, demand is estimated on base of the individual choice of each representative consumer from different demographics. Rasmusen (2007) concludes that the BLP method estimates the importance of the product characteristics, consumer characteristics, and prices using the generalized method of moments. This is a highly flexible method, requiring weaker assumptions than maximum of likelihood but like that procedure requiring a large number of observations and much computing power.

The completely different approach is discussed by Traverso and Abbas (2009) in deriving demand curve instead of the common curve fitting techniques. In particular, they propose a probability distribution that is assigned over the space of all functions that would satisfy the regularity conditions. As a result their method fundamentally differs from curve fitting since a probability is assigned over a full space of functions instead of assuming a strict functional form for the demand curve. They claim that this predictive model is compatible with rational decision making; it can be used to determine not only optimal pricing strategies, but also optimal information gathering strategies.

Leff (1975) in his research proposed that it can be profitable to set low price and to produce at high volume level, so that business will benefit high profit and society will benefit from cheaper prices. He approached to the demand curve as the horizontal summation of demand curves of consumer groups separated by the amount of wealth. He suggests that the prices are set optimally in the region where most consumers can benefit from the product. If the price is set a bit lower, then more consumers can be reached. In other words the shape of aggregate demand curve can tell us that more profit can be obtained by setting different price(s). The total consumer group of the product can be viewed as the separate groups of consumers by their purchasing power.

In the current paper we suggest new approach to aggregate demand analysis.

2. Definitions of the problems

We start with definition of basic concepts used in the article.

Observed Demand D – measured demand.

Smoothed Demand D_s – demand obtained by estimating of regression equation (possibly nonlinear), on the base of Observed Demand D

$$D=f(P). \quad (1)$$

Elementary Observed Demand D_i (i=1,2,….,r) – observed demands, the sum of which is equal (but not necessary!) to the Observed Demand D.

Smoothed Elementary Demands D_{si} (i=1,2,…,r) – which are represented by the regression equations (we assume it is linear, unlike to nonlinear equations of the Smoothed Demand D_s), based on Observed Elementary Demands D_i.

Aggregate Demand $D_{A\Sigma}$ – the vertical sum of the Smoothed Elementary Demands Dsi. DAΣ is the result of smoothing (by the vertical summation) of Observed Demand D.

It is assumed that the Observed Demand is

$$D = DA\Sigma + \varepsilon DA. \quad (2)$$

We consider two different representations of Observed Demand D: representation by regression model (1) and representation by Aggregate Demand DAΣ (2).

Above given definitions allows to formulate two mutually inverse related problems.

Direct problem. Given Smoothed Elementary Demands D_{si}, that is the set of parameters of d_i, it is required to define Smoothed Demand D_s, which assumed to be equal to aggregate values of $D_{A\Sigma}$. It is clear that the problem can be simply solved by calculating the value of $D_{A\Sigma}(Z_i)$ for the given relative price z_i.

Inverse problem. Given the Demand D_i for some values of z_i (i=1,….,n) and for end-prices x_i (i=1,….,r) on the interval of (0,x_{max}=1). It is required to define values of di that are Smoothed Elementary Demands D_{si}.

Usage of concept of aggregate demand can be justified as follows. We consider hypothetical data which clearly shows difference between direct usage of conventional regression analysis and approach based on the aggregate demand model. In fig.1 the observations of three independent elementary demands are given. They can be represented by means of linear regression equations which are shown as straight dashed lines in fig.1.

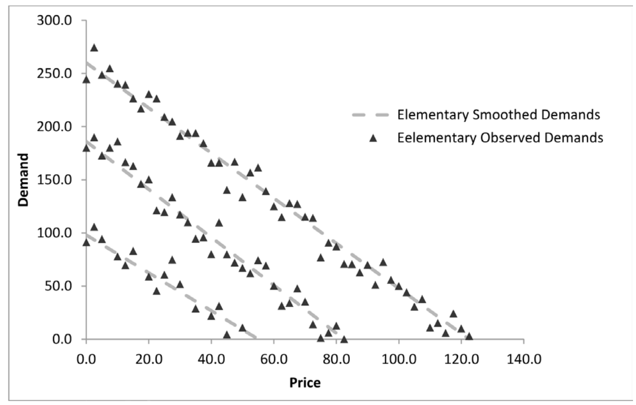

Figure 1. *Elementary Observed Demands and their Regression Lines.*

The three clouds of elementary demand observations can be represented (by means of summation) as one cloud of demand which is shown in fig.2. Direct application of conventional regression analysis to the data of fig.2 (observation are approximated by means of second order polynomial) does not permit to discover the hidden fact that the data was actually the result of interaction of three elementary demands.

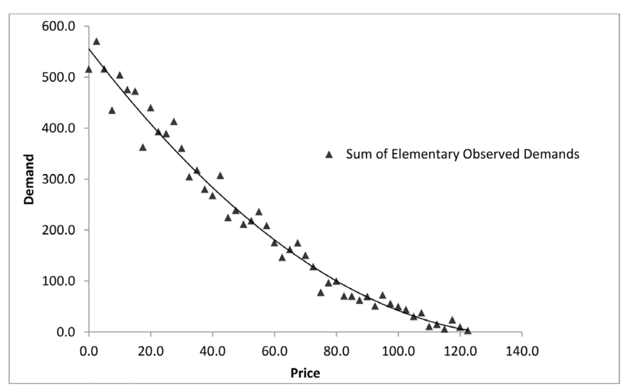

Figure 2. *Result of Summation of Three Elementary Demands.*

This very important fact completely falls out of frame of conventional regression analysis, whereas approach based on aggregate model permits to explicit hidden internal phenomena of Demand-Price compound interaction.

The above mentioned justifies importance of elabo-

1. Traditionally in economics literature the coordinate system is Price-to-Demand is used, whereas hereafter we shall use opposite Demand-to-Price system.

ration of mathematical methods of analysis of aggregate demands.

3. Basic Part

3.1.Theoretical Basics

The Fig.1 represents the general case of aggregative and elementary demands structure.

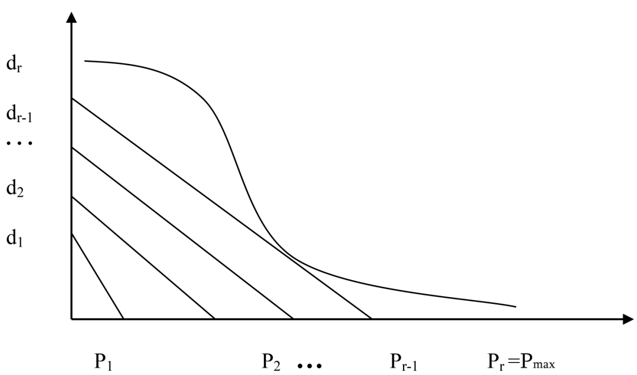

Figure 1. *Graphical Representation of Aggregate Demand DAΣ.*

The solid curve is the smoothed nonlinear demand Ds, straight lines are the smoothed elementary demands Dsi.

P_i – prices for the aggregate, where smoothed elementary demands D_{si} equals to zero, i.e. they are end-prices of smoothed elementary demands D_{si}. They may be equally spaced, or they may not be. Further we introduce unitless prices:

$$x_i = \frac{P_i}{P_{\max}} . \qquad (3)$$

P_{max} – where the price of nonlinear demand curve is equal to zero.

It is obvious from this expression that $0 \le x \le 1$ and x_{max}. Besides we introduce one more variable z=x (also unitless price), which will be used for technical purposes in integration.

Let us define the aggregate demand $D_{A\Sigma}$. The aggregate demand, being the summation of elementary demands, can be represented as follows:

$$D_{A\Sigma}(P) = \sum_{i=i_0}^{r} (d_i + a_i P) , \qquad (4)$$

where:

i_0 – value of index that is equal to the index of Pi which satisfies ;

r – number of elementary demands

Let $a_i = \dfrac{d_i}{P_i}$, then the equation (4) can be rewritten as:

$$D_{A\Sigma}(P) = \sum_{i=i_0}^{r} d_i (1 - \frac{P}{P_i}) \qquad (5)$$

Or, using the unitless price it can be rewritten as:

$$D_{A\Sigma}(z) = \sum_{i=i_0}^{r} d_i (1 - \frac{z}{x_i}) \qquad (6)$$

All three equations are representations of the Aggregate Demand $D_{A\Sigma}$. They are equivalent to each other, differing only in the written form, and the last one is more preferable.

Let us discuss the equation (6). Firstly, lower limit of summation is the variable value, depending on z (x and z are represent the relative price, in mathematical terms they are different: x_i is the end-prices, z is the current price from interval of (0, z_{max}=1)). Secondly, end-prices are fixed known values. Thirdly, as unknowns one can consider both $D_{A\Sigma}$ and d_i.

Assume that there are r intervals $\Delta_1=(0, x_1)$, $\Delta_i=(x_i\text{-}1, x_i)$,…,$\Delta r=(x_{r-1}, x_r=1)$ (note that $x_r=x_{max}=1$) set by r values of the end-prices, and let us suppose that values of z_j (j=1,…,n) distributed in the way that at least one of them falls into one of the intervals Δ_i. Let us define the quantity of z_j points, falling into i_{th} interval, through k_i. It is obvious that values of k_i should satisfy the equality $\sum_{i=1}^{r} k_i = n$. Thus, there are r groups of z_j points randomly distributed in intervals of Δ_i with one condition: "at least one of them falls into one of the intervals Δ_i". Note that some of them may coincide with the end-prices. Therefore there are r unknowns of di that requires defining $n \ge r$ values of z_j, and , that results in a system of linear equations with matrix :

$$D(z_j) = A_{ji} d_i . \text{ (i=1,…,r; j=1,…,n)} \qquad (7)$$

which is rectangular for n>r and square for n=r.

It is not difficult to conclude that matrix A of system (8) for n>r will have the following form:

$$A = \begin{vmatrix} (1-\frac{z_1}{x_1}) & ... & (1-\frac{z_1}{x_i}) & ... & (1-\frac{z_1}{x_r}) \\ ... & ... & ... & ... & ... \\ (1-\frac{z_{k_1}}{x_1}) & ... & (1-\frac{z_{k_1}}{x_i}) & ... & (1-\frac{z_{k_1}}{x_r}) \\ ... & ... & ... & ... & ... \\ 0 & ... & (1-\frac{z_{k_{i-1}+1}}{x_i}) & ... & (1-\frac{z_{k_{i-1}+1}}{x_r}) \\ 0 & ... & (1-\frac{z_{k_i}}{x_i}) & ... & (1-\frac{z_{k_i}}{x_i}) \\ ... & ... & ... & ... & ... \\ 0 & ... & 0 & ... & (1-\frac{z_{k_{r-1}+1}}{x_r}) \\ ... & ... & ... & ... & ... \\ 0 & ... & 0 & ... & (1-\frac{z_{k_r}}{x_r}) \end{vmatrix} \qquad (8)$$

and for n=r:

$$A = \begin{vmatrix} (1-\frac{z_1}{x_1}) & \dots & (1-\frac{z_1}{x_{r-1}}) & (1-\frac{z_1}{x_r}) \\ 0 & (1-\frac{z_2}{x_2}) & \dots & (1-\frac{z_2}{x_r}) \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & (1-\frac{z_r}{x_r}) \end{vmatrix} \quad (9)$$

In the first case we have an overdetermined system of n × r (the number of equations exceeds the number of unknowns), and the ordinary least squares method should be used to determine the di, while the second case is a system of linear equations r × r.

The solution of the second case does not present difficulties and leads to the solution of the system (7) with the matrix (9), which in this case is triangular, so that the solution even can be written out analytically without employing numerical calculations.

The first case, when there is overdetermined system (the number of equations exceeds the number of unknowns), requires the use of the ordinary least squares method.

It should be noted that we are modeling the dependence of Demand from Price, so that we are dealing with the function of one variable D=f(P), but system (7) represents this dependence as the function of r variables: z_i (i=1,…,r). Besides in system (7) instead of zi their functions - $(1-\frac{z_i}{x_j})$ are used, which for j=(1,…,n) create, referring to the language of regression analysis, matrix of observations of A of size n × r that are represented in (8). The estimated parameters are the values of di (i = 1, ..., r) that are the intercepts of elementary demands. Thus, the problem of estimating the aggregate is represented as a linear regression problem for r variables.

3.2. Regression Model of Aggregate Demand

Consider that the data of demand price observations are given in fig.4. Assume that on the base of economical, sociological, etc. information we defined three intervals of the prices. Now we can apply the elaborated approach to try to define three elementary demands components in initial set of the data. Because there are three defined intervals of prices, r=3. The latter means that to estimate elementary demands parameters we have to use linear regression method for three dummy variables. To form a matrix of dummy variables we elaborated special program in MatLab programming language (Sayfullin, 2011).

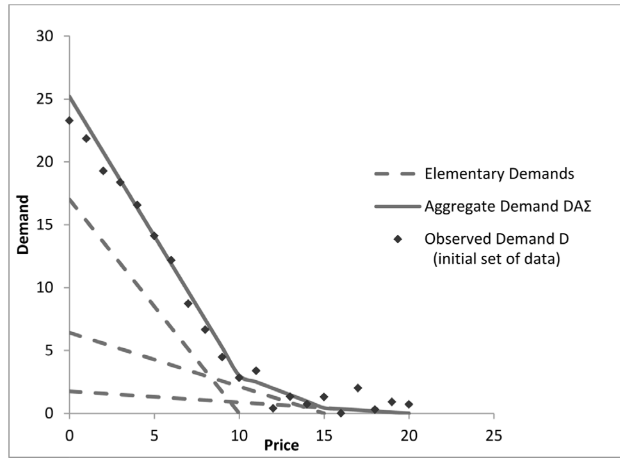

Figure 4. *Observed Demand D, Elementary Demands and Aggregate Demand $D_{A\Sigma}$*

The result of the application of the program of identification of parameters of elementary demand regression lines are shown in Table 1.

Parameters	Elementary Demand 1	Elementary Demand 2	Elementary Demand 3
Slope	-1.70	-0.43	-0.09
Intercepts	17.01	6.41	1.75

Table 1. *Estimated Parameters of Elementary Demands.*

One can see that observations represented in fig. 4 can be explained in terms of elementary demands which play role of components of aggregate demand. Clear that second order polynomial can also be applied to the data (fig. 5).

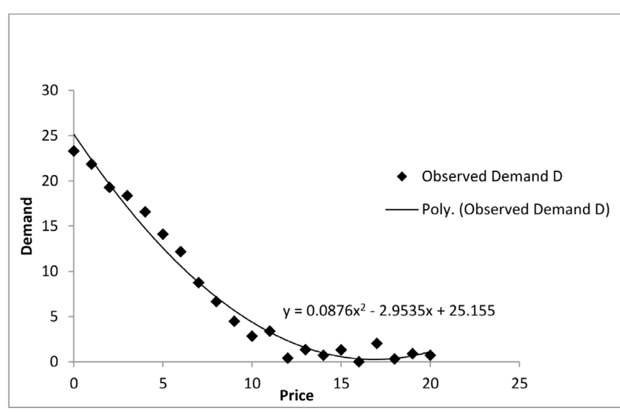

Figure 5. *Second Order Polynomial Approximation of Observed Demand D.*

The later approximation can be efficiently used for prediction and other computations connected with demand-price analysis, but it does not permit, as it was mentioned above, to unfold internal structure of the demand under consideration and to show compound nature of the demand.

Aggregate analysis gives the opportunity to view the contribution of the groups of the customers to shape of the market demand curve, the effect of the current price on the profit of the firm, and price adjustment suggestions.

Conclusion

A new method of analysis of demand internal structure and compound nature depended on the contribution of various groups of customers is suggested. New conceptions of Observed Demand D, Smoothed Demand Ds, Elementary Observed Demand Di, Smoothed Elementary Demands Dsi, and Aggregate Demand DAΣ are introduced. Direct and inverse problem of estimation of elementary and aggregate demands parameters are defined. Aggregate demand structure is represented as a multidimensional dummy variables regression model. The theoretical results are verified by means of corresponding numerical example.

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